# Measurement Kata Workbook

\*\*What is this workbook?\*\* A workbook is a collection of problems, accompanied by solutions to them. The explanations focus on the logical steps required to solve a problem; they illustrate the concepts that need to be applied to come up with a solution to the problem, explaining the mathematical steps required.

Note that a workbook should not be the primary source of knowledge on the subject matter; it assumes that you've already read a tutorial or a textbook and that you are now seeking to improve your problem-solving skills. You should attempt solving the tasks of the respective kata first and turn to the workbook only if stuck. While a textbook emphasizes knowledge acquisition, a workbook emphasizes skill acquisition.

This workbook describes the solutions to the problems offered in the [Measurement Kata](https://notebooks.gesis.org/binder/jupyter/user/microsoft-quantumkatas-iacsymjt/notebooks/Measurements/Measurements.ipynb). Since the tasks are offered as programming problems, the explanations also cover some elements of Q# that might be non-obvious for a first-time user.

\*\*What you should know for this workbook?\*\*

You should be familiar with the following concepts before tackling the Measurements kata (and this workbook):

1. Basic linear algebra

2. The concept of qubit and multi-qubit systems

3. Single-qubit and multi-qubit quantum gates

4. Superposition

## \*\*Part I. Discriminating Orthogonal States\*\*

### \*\*Task 1.1. |0⟩ or |1⟩ ?\*\*

\*\*Input:\*\* A qubit which is guaranteed to be in either the |0⟩|0⟩ or the |1⟩|1⟩ state.

\*\*Output:\*\* true if the qubit was in the |1⟩ state, or false if it was in the |0⟩ state. The state of the qubit at the end of the operation does not matter.

### \*\*Solution\*\*

operation IsQubitOne (q : Qubit) : Bool {

return (M(q) = One);

}

When a qubit is measured its superposition state collapses, meaning that its value goes from probabilistic (how likely it is to be 1 or 0) to determinist (0 or 1). Since the qubit can only be measured as 0 or 1 we need a method to determine which one it is.

[M( )](https://docs.microsoft.com/en-us/qsharp/api/qsharp/microsoft.quantum.primitive.m?view=qsharp-preview) is a Q# method which returns true if the qubit is in state 1 and false if the qubit is in state 0.

Here we evaluate M(q) and test if it is equal to 1.

### \*\*Task 1.2. Set the qubit to the |0⟩ state.\*\*

\*\*Input:\*\* A qubit in an arbitrary state.

\*\*Goal:\*\* Change the state of the qubit to |0⟩|0⟩.

### \*\*Solution\*\*

operation InitializeQubit (q : Qubit) : Unit {

if(M(q) = One){

X(q);

}

}

Here we want a method that always returns a qubit in state |0⟩. Since when we measure the qubit can only be in state 0 or in state 1 we have two possibilities:

1. The qubit is in state |0⟩, we don’t need to change anything

2. The qubit is in state |1⟩, we need to inverse the state of the qubit

How to inverse the state of a qubit?

We use the [X()](https://docs.microsoft.com/en-us/qsharp/api/qsharp/microsoft.quantum.primitive.x?view=qsharp-preview) gate

![X gate matrix representation](https://imgur.com/a/uFGDngU).

Recall that |0⟩ can be represented as ![0 vector representation](https://imgur.com/h4csF4Y) and |1⟩ can be represented as ![1 vector representation](https://imgur.com/undefined)

- When we apply the X gate to |0⟩ we obtain |1⟩

![X(|0⟩) = |1⟩](https://imgur.com/1hVgMD7)

- When we apply the X gate to |1⟩ we obtain |0⟩

![X(|1⟩) = |0⟩](https://imgur.com/IzYxMNZ)

Therefor if the qubit is |1⟩ and we apply the X( ) gate to change its state to |0⟩.

### \*\*Task 1.3. |+⟩ or |−⟩?\*\*

\*\*Input:\*\* A qubit which is guaranteed to be in either the |+⟩ or the |−⟩ state. As a reminder, |+⟩=1/2√(|0⟩+|1⟩), |−⟩=1/2√(|0⟩−|1⟩).

\*\*Output:\*\* true if the qubit was in the |+⟩ state, or false if it was in the |−⟩ state. The state of the qubit at the end of the operation does not matter.

### \*\*Solution\*\*

operation IsQubitPlus (q : Qubit) : Bool {

H(q);

if (M(q) == Zero){

return true;

}

else{

return false;

}

}

In the Superposition Kata we were introduced to the Hadmard gate which converts |0⟩ -> |+⟩ and |1⟩ -> |−⟩

Since the qubit can be either |0⟩ (or |+⟩) or |1⟩ (or |−⟩) we have two possibilities:

1. The qubit is 1 (or |−⟩) we return false

2. The qubit is 0 (or |+⟩) we return true

### \*\*Task 1.4. |𝐴⟩ or |𝐵⟩?\*\*

\*\*Inputs:\*\*

1. Angle 𝛼α, in radians, represented as a Double.

2. A qubit which is guaranteed to be in either the |𝐴⟩ or the |𝐵⟩ state, where |𝐴⟩=cos𝛼|0⟩+sin𝛼|1⟩ and |𝐵⟩=−sin𝛼|0⟩+cos𝛼|1⟩.

\*\*Output:\*\* true if the qubit was in the |𝐴⟩ state, or false if it was in the |𝐵⟩ state. The state of the qubit at the end of the operation does not matter.

### \*\*Solution\*\*

operation IsQubitA (alpha : Double, q : Qubit) : Bool {

Ry(-2.0 \* alpha, q);

return M(q) == Zero;

}

The [Ry](https://docs.microsoft.com/en-us/qsharp/api/qsharp/microsoft.quantum.primitive.ry?view=qsharp-preview) gate rotates a single qubit q through an angle 𝛼α around the y-axis.

![Ry gate matrix representation](https://imgur.com/undefined)

In this method we first rotate the qubit q through an angle of 𝛼α using the Ry operation.

Follows two possibilities:

1. The qubit is mesured as |1⟩, the qubit is in state |𝐵⟩, we return false.

2. The qubit is mesured as |0⟩, the qubit is in state |𝐴⟩, we return true.

### \*\*Task 1.5. |00⟩ or |11⟩ ?\*\*

\*\*Input:\*\* Two qubits (stored in an array of length 2) which are guaranteed to be in either the |00⟩ or the |0⟩ state.

\*\*Output:\*\* 0 if the qubits were in the |00⟩ state, or 1 if they were in the |11⟩ state. The state of the qubits at the end of the operation does not matter.

### \*\*Solution\*\*

operation ZeroZeroOrOneOne (qs : Qubit[]) : Int {

if(M(qs[0]== One){

return One;

}

else{

return Zero;

}

}

Both of the qubits stored in the qs array are in the same state ( for |00⟩ each individual qubit is in state |0⟩, for |1⟩ each individual qubit is in state |1⟩).Therefore, if we measure one qubit we will know the state of the other.

In other words, if M(qs[0]) measures the first qubit as 1 we know that the qubits in qs are in state |1⟩ and M(qs[0]) measures the first qubit as 0 we know that the qubits in qs are in state |00⟩.

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